

Responding to Coding errors made by NZCC

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Table of contents

1	Introduction	2
2	Summary of errors	2
3	Error #1 – incorrect syntax in averaging weekly and four-weekly asset betas.....	3
4	Error #2 – incorrect implementation of the weighted regression	4
4.1	The “Flint method” is a WLS regression	5
4.2	NZCC’s incorrect, and corrected, implementation of the WLS regression	5
4.3	Alternative correction without using the built-in WLS function in R.....	7

List of figures

Figure 3-1: Reproduction of the Table 4.4 in the final decision.....	3
Figure 3-2: Screenshot of the relevant codes with errors in “Master_Airports.r”	3
Figure 3-4: Screenshot of the corrected codes in reference to Figure 3-2.....	4
Figure 3-5: Illustration of the “mean()” function in R	4
Figure 4-1: Correction to NZCC’s code using the weekly estimation’s code	6
Figure 4-2: Alternative correction to NZCC’s code	8
Figure A-1: Reproduction of Dr. Lally’s equation 2	8
Figure A-2: Example of econometric textbook on the WLS model	10

1 Introduction

1. In its final decision, the NZCC published the complete R code used in its calculations for the asset beta.¹ Some of this R code was not available earlier in the process due to significant changes between the draft and the final decision.
2. Notably, the methodology in calculating the pandemic adjustment has changed substantially including in relation to the NZCC's application of the "Flint method".
3. After reviewing the final decision and the accompanied R code, we have identified two unambiguous coding errors that were not apparent earlier in the process. This report sets out those errors and explains how they can be corrected. Consistent with our instructions, we do not opine on matters where we consider that the Commission has exercised its judgement inappropriately and/or inconsistently. Failure to raise such matters in this report should not be interpreted as implying that the experts do not consider any such matters exist.
4. We confirm that we have been referred to the Code of Conduct for Expert Witnesses (Code), as contained in Schedule 4 of the High Court Rules 2016 for New Zealand, and that this report has been prepared in accordance with that Code.
5. Subject to the following exception, this report expresses the joint opinions of all experts. The exception is that Mr Balchin has not independently re-run the Commission's R-code with the corrections made, and so is not expressing an opinion on those matters.

2 Summary of errors

6. This report outlines two unambiguous coding errors in the NZCC R code used to derive asset beta values consistent with the reasoning in the 2023 IM final decision.
7. **Error#1.** The NZCC intended to calculate the "pre-COVID" asset beta by averaging the weekly and four-weekly pre-COVID asset betas. However, the code only averaged the weekly asset betas. This correction will change the pre-COVID asset beta from 0.63 to 0.65.
8. **Error #2.** The NZCC sought to apply the "Flint method" to inform its judgment in determining an appropriate level of pandemic adjustment. The Flint method was to apply a Weighted Least Square (WLS) regression for calculating equity betas (which are then transformed into asset betas) where COVID affected data is given less weight than non-COVID affected data in line with an assumed lower *ex ante* probability of a future pandemic. This use of WLS was also explicitly recommended by TDB and Dr. Lally.
9. The NZCC code incorrectly implemented the WLS which resulted in the regression coefficients having no meaningful economic interpretation. The closest interpretation possible is that the regression is actually modelling a one in 660 year pandemic when it is trying to model a one in 50 year pandemic. The correct implementation of the WLS regression changes the range of the pandemic premium for AIAL's weekly asset beta from the reported 0.02-0.08 range in the final decision to 0.07-0.15.

¹ The NZCC published a .zip file containing the R code that it has used to calculate its asset beta and leverage estimates based on data extracted from Bloomberg, along with a covering note that indicates how the R code should be run. See: NZCC, Part 4 IM Review 2023 Cost of capital topic paper asset beta calculations – R code covering note, 13 December 2023. NZCC, Part 4 IM Review 2023 Cost of capital topic paper - Asset beta calculations, Zip file, 13 December 2023.

3 Error #1 – incorrect syntax in averaging weekly and four-weekly asset betas

10. The intention of the final decision was to estimate the pre-COVID asset beta as the average weekly and four-weekly asset beta estimates. For example, the first row of Table 4.4 states that weekly and four-weekly asset betas are included in the averaging. However, the result in that same row for the final decision should be 0.65 instead of 0.63 (where 0.63 is the average of weekly asset betas only).²

Figure 3-1: Reproduction of the Table 4.4 in the final decision

145

Table 4.4 Data to inform the determination of the airport equity beta and leverage

Indicator	Current IMs	draft decision	Inputs for final decision
Pre-Covid asset beta (long term average, weekly and four-weekly)		0.53	0.63
Pandemic premium based on our adjustment of TDB Advisory's calculation applied to Auckland Airport		0 – 0.04	0.02 – 0.08
Pandemic-adjusted asset beta		0.53 – 0.57	0.65 – 0.71
Asset beta for year to 30 September 2022 (weekly and four-weekly, including Hainan)		0.50	0.72
Asset beta for year to 30 September 2022 (weekly and four-weekly, excluding Hainan)		0.50	0.61
Asset beta for last two five-year periods (weekly and four-weekly)	0.65	0.63	0.74
Adjustment to asset beta	-0.05	0	0
Final asset beta	0.60	0.55	0.67

Source: "Cost of capital topic paper, Part 4 Input Methodologies Review 2023 – Final decision", p.145.

11. The relevant errors are found in line 256 to 263 of the NZCC's published "Master_Airports.r" R code.³ Below is a screenshot of the relevant code for the calculation of asset betas:

Figure 3-2: Screenshot of the relevant codes with errors in "Master_Airports.r"

```

256 #Calculate the pre-Covid asset beta
257
258 PreCovid1A = mean(weeklyP1A.mean, FourweeklyP1A.mean) * 1826/4533
259 PreCovid2A = mean(weeklyP2A.mean, FourweeklyP1A.mean) * 1826/4533
260 PreCovid3A = mean(weeklyP4A.mean, FourweeklyP4A.mean) * 880/4533
261
262 PreCovidA = sum(PreCovid1A, PreCovid2A, PreCovid3A)
263 PreCovidA

```

Source: \Airport analysis\ Master_Airports.r, line 256 to 263.

² Cross-check can be made using Table D2 in the final decision, where the average weekly and four-weekly asset betas for 2007-2012, 2012-2017 and 2018-2020 are 0.69, 0.6 and 0.65 respectively. The weighted average of the three: $0.69 \times \frac{5}{12} + 0.6 \times \frac{5}{12} + 0.65 \times \frac{2}{12} \approx 0.65$.

³ For completeness, similar errors are also found in the calculation of pre-COVID equity beta and leverage in line 265 to 296. Note that NZCC do not currently rely on these numbers in their final decision.

12. It is apparent that, consistent with the wording of the final decision, the intention of the code is to average between weekly and four-weekly asset betas as both are named in rows 258 to 260. However, since the “mean()” function in R only averages the vector provided in the first input, it will therefore only average the weekly number (“WeeklyP1A.mean” for line 258).
13. Furthermore, the second element of line 259 should be “FourWeeklyP2A.mean” (which is the 2012-17 period) instead of “FourWeeklyP1A.mean” (which is the 2007-12 period).
14. Finally, the number of days in the first period (30 September 2007 to 1 October 2012) should be 1827, and in the third period (1 October 2017 to 28 February 2020) should be 881. The total number of days for the three periods should be 4534.
15. The corrected code is provided in the screenshot below, with red lines showing the changes.⁴

Figure 3-3: Screenshot of the corrected codes in reference to Figure 3-2

```

266 PreCovid1A = mean(c(weeklyP1A.mean, FourweeklyP1A.mean)) * 1827/4534
267 PreCovid2A = mean(c(weeklyP2A.mean, FourweeklyP2A.mean)) * 1826/4534
268 PreCovid3A = mean(c(weeklyP4A.mean, FourweeklyP4A.mean)) * 881/4534
269
270 PreCovidA = sum(PreCovid1A, PreCovid2A, PreCovid3A)
271 PreCovidA
  
```

16. The “c()” function combines the weekly and four-weekly elements into one vector for the “mean()” function to evaluate them correctly. After correcting the error, the pre-COVID asset beta increases from 0.63 to 0.65.
17. In the figure below, we illustrate an error from the NZCC's published code. The left screenshot shows the incorrect use of the "mean()" function, returning only the first input. The right screenshot demonstrates the correct use, where the "mean()" function accurately returns the average after the numbers are inputted into the "c()" function first.

Figure 3-4: Illustration of the “mean()” function in R

<u>Incorrect use of the mean() function</u>	<u>Correct use of the mean() function</u>
<pre>> mean(10, 90) [1] 10</pre>	<pre>> mean(c(10, 90)) [1] 50</pre>

4 Error #2 – incorrect implementation of the weighted regression

18. The final decision sought to apply the “Flint method” to inform the NZCC’s judgment in determining an appropriate level of pandemic adjustment.⁵ The Flint method requires the application of a weighted least square (WLS) regression. The NZCC code failed to correctly apply a WLS regression. Correctly implementing the WLS regression, the range of the pandemic premium for AIAL’s weekly asset beta increases from 0.02-0.08 to 0.07-0.15.
19. This section demonstrates:
 - a. That the Flint method requires application of a WLS regression.;

⁴ Once again, similar corrections should also be applied to the pre-COVID equity beta and leverage.

⁵ Final decision paragraph 4.187 to 4.204.

- b. That the NZCC's code did not correctly apply a WLS regression; and
- c. The required correction to the NZCC code.

4.1 The “Flint method” is a WLS regression

20. The WLS regression is the means for integrating both weights and linear regression into a single model.

21. Flint is clear that it ran a weighted least squares regression.⁶

To estimate the COVID adjustment, we have developed a methodology which creates a ‘reweighted’ beta estimate based on daily share price and index data over recent years...We then calculate an equity beta for each comparator using a linear regression, with different weights assigned to COVID and non-COVID observations. The weights can be translated – in effect – into an equivalent ‘frequency’ at which a ‘COVID-like’ event occurs.

22. The UK Civil Aviation Authority’s response to notices of appeal against its final decision for Heathrow Airport similarly describes Flint’s approach as a weighted least squares estimate:⁷

*Secondly, it is said that the CAA and its advisors Flint Global were wrong to rely on a **weighted least squares (“WLS”) estimator** ... As to this, it was reasonable for the CAA to have used the **WLS estimator**...*

23. In its final determination on the Heathrow Airport appeals, the UK Competition and Markets Authority also describes Flint’s approach as a weighted least squares estimate:⁸

*In this context, our view is that the **CAA/Flint approach (utilising WLS rather than OLS)**, was not materially different to recent regulatory practice, with the only difference being that the CAA **re-weighted pandemic datapoints** to capture its view that the likelihood of future pandemics will be lower than observed in the historical data set.*

24. TDB stated that they intended to give effect to the “Flint method” by applying a WLS regression.⁹

*Hence, rather than estimating the equity betas using a simple linear regression, we consider a **weighted least squares regression model**. This method allows for certain observations to be given more weight or influence in the regression model.*

25. Similarly, Dr. Lally advised the NZCC that the Flint method (and TDB’s description of its own method) was to apply a WLS regression. Dr Lally also recommended the use of the Flint/WLS regression method. We note that the WLS regression is equivalent to equation 2 of Dr. Lally’s report.¹⁰

4.2 NZCC’s incorrect, and corrected, implementation of the WLS regression

26. In its R code, the NZCC attempts to implement a WLS regression by transforming the underlying data and then applying an ordinary least squares (OLS) regression on the transformed data. It is possible to give effect to a WLS regression by running an OLS regression on transformed data. However, the

⁶ Flint August 2021, Support to the Civil Aviation Authority: Estimating Heathrow’s beta post-COVID-19, p.12 – 14.

⁷ Civil Aviation Authority, Before the Competition and Markets Authority in the matter of an appeal under section 25 of the Civil Aviation Act 2012 between Heathrow Airport Limited, British Airways PLC, Virgin Atlantic Airways Limited, Delta Air Lines Inc (Appellants) and the Civil Aviation Authority (Respondent), Response to appeal | Non-confidential version, para 154.

⁸ Competition and Markets Authority, H7 Heathrow Airport licence modification appeals, Final determinations, 17 October 2023, para 6.152.

⁹ TDB, Auckland Airport’s Asset Beta: Covid-19 Adjustment Using Flint Study, 26 January 2023, p. 5.

¹⁰ The mathematical representation of the WLS model is detailed in 0.

NZCC did not correctly transform the data (we discuss this in greater detail in section 4.3 and Appendix A), which in turn results in incorrect estimates of the pandemic premium.

27. This section outlines a simpler implementation of the WLS regression in R.
28. The incorrect implementation of the Flint method can be found in the NZCC R code “Flint analysis – Airports” folder. The underlying calculations and relevant lines of code are:
 - a. Line 118 to 123 in “Estimate_daily.r”;
 - b. Line 124 to 129 in “Estimate_weekly.r”, and;
 - c. Line 155 to 160 in “Estimate_fourWeekly.r”.
29. Figure 4-1 below uses “Estimate_weekly.r” as an example.¹¹ The first red underline at line 130 is the incorrect regression where the NZCC is running an OLS regression of the weighted return data (defined in lines 124 and 125).¹²

Figure 4-1: Correction to NZCC’s code using the weekly estimation’s code

```

124 Data$WIndex <- Data$INDEX_PX_LAST_weekly_change * Data$weight
125 Data$WPrice <- Data$PX_LAST_weekly_change * Data$weight
126
127 #Using a groupwise operation (individual day of week regressions)
128
129 RegressionResults <- Data %>% group_by(Code, weekday) %>%
130   do(model = lm(WPrice ~ WIndex, data = .)) #NZCC's model (with modelling error)
131
132 RegressionResults <- Data %>% group_by(Code, weekday) %>%
133   do(model = lm(PX_LAST_weekly_change ~ INDEX_PX_LAST_weekly_change, data = ., weights = weight)) #Correct modelling
134

```

30. The correct version of the code is provided in line 133 (also red underline). This uses the R instruction to turn the “lm()” regression function,¹³ from an OLS regression into a WLS regression. This is achieved by including “, weights =” at the end of the “lm()” function and then specifying the desired weights after the equals sign. Both the dependent and independent variables used are the untransformed return observations. Line 133 differs from the NZCC’s line 130 as follows:
 - a. Line 133 uses the underlying (untransformed) return observations (not the weighted return observations defined in lines 124 and 125 and used in line 130); and
 - b. At the end of the regression “lm()” instruction, there is added “, weights = weight”. This instruction can be broken down as follows.
 - i. The “, weights” component of this specifies that the regression to be performed is a weighted regression. This is how a user instructs R to run a WLS regression;
 - ii. The “= weight” component of the addition specifies that the weights are to be those defined by the “Data\$weight” variable which has been specified elsewhere by the NZCC in the R code.¹⁴
31. The correct R code should be replicated for daily and four-weekly applications of the Flint method.

¹¹ The daily and four-weekly version of their respective file have the same incorrect syntax, which can be corrected with the same modification.

¹² In line 124 and line 125 the Commission defines new variables that transform the non-COVID stock and index price changes by the elsewhere defined “Data\$weight” variable. Line 124 calculates the index price change “Data\$INDEX_PX_Last_weekly_change” multiplied by the relevant weights. Line 125 calculates the airport stock price change multiplied by the relevant weights (“Data\$weight”). Noting that “weight” is the name of the variable defined by the NZCC elsewhere and the prefix “Data\$” tells R that what comes after is the name of a variable from the table “Data”. In this case, “weight” is a vector defined to be a value greater than 1 for “non-COVID” data and equal to 1 for COVID data. This is defined in line 121 to 122 in the weekly estimation code.

¹³ The “lm” function is the most common function in R for researchers to perform regression analysis. It is in the “stats” package which is part of base R and does not require additional installation.

¹⁴ Note that “weight” is a user specified name, and it could just as easily have been termed “weights_for_regression” or any other name.

32. Correctly implementing the WLS model, the range of the pandemic premium for AIAL's weekly asset beta should change from 0.02-0.08 to 0.07-0.15.

4.3 Alternative correction without using the built-in WLS function in R

33. An alternative to using R's built in WLS function is to implement a WLS regression by transforming the underlying data and then applying an OLS regression on the transformed data.
34. This is the method that the NZCC was attempting to implement in lines 124 to 130 of the NZCC code shown in Figure 4-1. In those lines, the NZCC code multiplies the return data by the desired weight, then performs an OLS regression using the transformed (weighted) return data.
35. It is possible to give effect to a WLS regression by running an OLS regression on transformed data. However, in order to do this correctly, and as set out in more detail in Appendix A:
- a. the data needs to be transformed by the square root of the desired weights. This is because the OLS regression minimises the sum of squared errors. Therefore, in order to "force" the OLS regression to give the desired weight to each observation that observation needs to be transformed by the square root of the desired weight; and
 - b. the column of weights needs to be applied to all elements of the equation that is being estimated via OLS, the effect of which is that:
 - i. the intercept variable (which is notionally a column comprising the value 1) is transformed into a column of weights – that is, the column of weights needs to be included as a new variable, and
 - ii. the transformed equation must then be estimated without an intercept variable (i.e., the OLS regression must be forced to pass through the origin).
36. The NZCC's code does not perform either step correctly:
- a. The NZCC transforms the data by the desired weights rather than the square root of the desired weights. Consequently, the actual weight the NZCC gives to the "non-COVID" observation is the square of the desired weights. This means, for example, when the NZCC is trying to model a one in 50 year pandemic it is actually modelling a one in 660 year pandemic.¹⁵
 - b. The NZCC does not include the weights as an independent variable alongside the transformed observations, but, instead, allowed the OLS regression to find its own (unweighted) intercept. This failure means that the NZCC regression effectively shifts COVID and non-COVID observations relative to each other rather than re-weighting them. The resulting regression estimate has no meaningful economic interpretation. The coefficients cannot be interpreted as a weighted beta (i.e., even with the incorrect square of the desired weights).
37. Transforming the non-COVID data by the square of the desired weights (point a. above) would, on its own, bias the estimated Flint adjustment towards zero. However, the failure to correctly estimate the weighted intercept (point b. above) results in a regression coefficient that has no meaningful economic interpretation.

¹⁵ Let the relevant scenario be a 17 month COVID period, a 5 year (60 month) regression period, and a hypothesised 17 month future pandemic occurring once every 50 years (600 months). The desired weight to the non-COVID data (assuming a weight of 1.0 to the COVID data) is $13.6 = (600-17)/(60-17)$. However, the actual weight applied by the NZCC is the square of 13.6 (=184). Consequently, the implied frequency of a pandemic is X months where $184 = (X-17)/(60-17)$. Solving for X gives 7,921 months (or 660 years).

38. Figure 4-2 below shows the correct code used to perform a WLS regression by transforming the data within an OLS regression instead of using the built in WLS function in R.

Figure 4-2: Alternative correction to NZCC’s code

```

131 Data$weight <- (Data$weight)^0.5
132 Data$wIndex <- Data$INDEX_PX_LAST_weekly_change * Data$weight
133 Data$wPrice <- Data$PX_LAST_weekly_change * Data$weight
134
135 #using a groupwise operation (individual day of week regressions)
136
137 RegressionResults <- Data %>% group_by(Code, weekday) %>% do(model = lm(wPrice ~ weight + wIndex + 0, data = .))
138

```

39. Line 131 ensures the square-root of the desired weight is used to transform the data. Line 137 includes the square-root of the desired weight as an independent variable replacing the OLS “constant”, which is removed by adding “+0” into the formula.
40. This gives the same results as the built-in WLS model described in section 4.2.

Appendix A Mathematical description of WLS and NZCC’s incorrect transformation

41. Dr Lally correctly describes the Flint method as being the same as his “equation 2”.¹⁶ Dr. Lally’s equation 2 expressed the pandemic adjusted beta as a weighted covariance between airport and market returns divided by a weighted variance of market returns (with the weights applied to COVID and non-COVID data). Dr Lally’s formula is for a special case where the regression has the same number of observations that are COVID affected and non-COVID affected (N observations of each, 2N in total).

Figure A-1: Reproduction of Dr. Lally’s equation 2

$$\beta = \frac{.075 \sum_{j=1}^N (R_{xj} - E_x)(R_{mj} - E_m) + .925 \sum_{j=N+1}^{2N} (R_{xj} - E_x)(R_{mj} - E_m)}{.075 \sum_{j=1}^N (R_{mj} - E_m)^2 + .925 \sum_{j=N+1}^{2N} (R_{mj} - E_m)^2} \quad (2)$$

Source: Dr Martin Lally, *The impact of future COVID-19 scenarios on beta* (22 June 2023), equation 2.

42. Two points can be made using equation (2). First, this is mathematically identical to the WLS regression with weights of 7.5% and 92.5%. A more generalised form of this formula (that allows for the possibility that not exactly half the dataset is pandemic affected and where the weighting scheme is more flexible with W_p = pandemic weight and W_{np} = non-pandemic weight) is provided below:

$$\beta_{WLS} = \frac{W_p \times (\text{Covariance pandemic data}) + W_{np} \times (\text{Covariance non - pandemic data})}{W_p \times (\text{Market variance pandemic data}) + W_{np} \times (\text{Market variance non - pandemic data})}$$

43. Secondly, Dr Lally expressed these weights outside (i.e., applied to) the covariance/variance statistics (in the numerator/denominator) of equation 2. However, if the weights were to be applied to the underlying data then, in order to give the same answer, the square root of the weights would need to be applied. This is because the correlation/variance statistics involve multiplying the deviation from the mean such that the weights applied to the observations will be multiplied together. That is:

¹⁶ Dr Martin Lally, *The impact of future COVID-19 scenarios on beta* (22 June 2023), p.5.

Equation 1: Generalised form of NZCC's approach without a constant

$$\beta_{WLS} = \frac{\sum_{j=1}^{j=N} w_j (R_{xj} - E_x)(R_{mj} - E_m)}{\sum_{j=1}^{j=N} w_j (R_{mj} - E_m)^2} = \frac{\sum_{j=1}^{j=N} (\sqrt{w_j} R_{xj} - \sqrt{w_j} E_x)(\sqrt{w_j} R_{mj} - \sqrt{w_j} E_m)}{\sum_{j=1}^{j=N} (\sqrt{w_j} R_{mj} - \sqrt{w_j} E_m)^2}$$

- N = the number of data points.
- Rx = the return on the stock.
- Rm = the return on the index.
- w_j = the weight assigned to each data point (in Dr Lally's example w_j would be 7.5% if the observation was deemed to be pandemic affected and 92.5% otherwise).
- E_x and E_m are the expected return on the stock and index respectively, estimated by $E_x = \frac{\sum_{i=1}^{i=N} w_i (R_{xi})}{\sum_{i=1}^{i=N} w_i}$ and $E_m = \frac{\sum_{i=1}^{i=N} w_i (R_{mi})}{\sum_{i=1}^{i=N} w_i}$ respectively.

44. Two important points can be made in applying Equation 1 to perform the WLS model using the transformation method (i.e., not using the built-in WLS function in R) as the NZCC attempted to do.
- a. It is necessary to use the square root of the desired weight, and;
 - b. The square root of the weight is also applied to both the expected return on the stock and index. Mathematically, these are constants that represent the intercept of the OLS regression. Therefore, a vector of the square root of the weight needs to be multiplied to the constant.
 - i. Programmatically, this is performed by forcing the OLS regression to pass through the origin (i.e., no intercept variable) and, instead, the WLS intercept must be estimated by including the weights themselves as an additional independent variable – with the OLS coefficient on the weights being the WLS intercept.
45. Note that the transformation method using the square root of the desired weight can also be found in econometric textbooks such as the one below.

Figure A-2: Example of econometric textbook on the WLS model

idea is that less weight is given to observations with a higher error variance; OLS gives each observation the same weight because it is best when the error variance is identical for all partitions of the population. Mathematically, the WLS estimators are the values of the b_j that make

$$\sum_{i=1}^n (y_i - b_0 - b_1x_{i1} - b_2x_{i2} - \dots - b_kx_{ik})^2/h_i \quad [8.27]$$

as small as possible. Bringing the square root of $1/h_i$ inside the squared residual shows that the weighted sum of squared residuals is identical to the sum of squared residuals in the transformed variables:

$$\sum_{i=1}^n (y_i^* - b_0x_{i0}^* - b_1x_{i1}^* - b_2x_{i2}^* - \dots - b_kx_{ik}^*)^2.$$

Since OLS minimizes the sum of squared residuals (regardless of the definitions of the dependent variable and independent variable), it follows that the WLS estimators that minimize (8.27) are simply the OLS estimators from (8.26). **Note carefully that the squared residuals in (8.27) are weighted by $1/h_i$, whereas the transformed variables in (8.26) are weighted by $1/\sqrt{h_i}$.**

A weighted least squares estimator can be defined for any set of positive weights. OLS is the special case that gives equal weight to all observations. The efficient procedure, GLS, weights each squared residual by the *inverse* of the conditional variance of u_i given \mathbf{x}_i .

Obtaining the transformed variables in equation (8.25) in order to manually perform weighted least squares can be tedious, and the chance of making mistakes is nontrivial. Fortunately, most modern regression packages have a feature for computing weighted least squares. Typically, along with the dependent and independent variables in the original model, we just specify the weighting function, $1/h_i$, appearing in (8.27). That is, we specify weights proportional to the inverse of the variance. In addition to making mistakes less likely, this forces us to interpret weighted least squares estimates in the original model. In fact, we can write out the estimated equation in the usual way. The estimates and standard errors will be different from OLS, but the way we *interpret* those estimates, standard errors, and test statistics is the same.

Source: Jeffrey M. Wooldridge, *Introductory Econometrics: A Modern Approach*, p.280-283.