THE IMPACT OF FUTURE COVID SCENARIOS ON BETA

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1. Introduction

In a recent draft report on WACC, the Commerce Commission (2023, pp. 70-74) concluded that covid-19 raised the betas of airports and there may be a material chance of a future event of that type, leading to them estimating the beta for a future regulatory period by placing weights on the beta estimates from the pre covid and covid periods, with the weights equal to the probabilities of these two scenarios arising in the future. The merits of applying such treatment to selected events are contentious, especially when the probability of a recurrence of the event is so hard to estimate and any such recurrences may be materially more or less severe. However, this paper focuses purely upon the mechanics of the adjustment used by the Commerce Commission.

2. Analysis

By definition, the true beta applicable to a future period for some asset (x) is the covariance between the asset return (R_x) and the market return (R_m) divided by the variance of the market return, where the returns R_x and Rm are each drawn from their respective future probability distributions with expectations of E_x and E_m respectively:
:

$$\beta = \frac{Cov(R_x, R_m)}{Var(R_m)} = \frac{E(R_x - E_x)(R_m - E_m)}{E(R_m - E_m)^2} \tag{1}$$

In the interests of presentational simplicity, and without loss of generality, suppose that there are N equally possible outcomes for R_x if a covid type scenario does not arise, and N equally possible outcomes for R_x if a covid scenario does arise. Also, and consistent with an example presented by the Commerce Commission (ibid, para 4.64), the probability of the covid scenario is 0.075 and that of the non-covid scenario is 0.925. Thus, there is a 7.5% chance of R_x being drawn from the covid distribution and then a $1/N$ chance of each outcome within that distribution. There is also a 92.5% chance of R_x being drawn from the non-covid distribution and then a $1/N$ chance of each outcome within that distribution. The same applies to R_m. Letting the covid scenario returns appear first in both the numerator and denominator, it follows that beta is

$$\beta = \frac{\frac{.075}{N}\sum_{j=1}^{j=N}\left(R_{xj} - E_x\right)\left(R_{mj} - E_m\right) + \frac{.925}{N}\sum_{j=N+1}^{j=2N}(R_{xj} - E_x)(R_{mj} - E_m)}{\frac{.075}{N}\sum_{j=1}^{j=N}(R_{mj} - E_m)^2 + \frac{.925}{N}\sum_{j=N+1}^{j=2N}(R_{mj} - E_m)^2} \qquad (2)$$

The question of interest is how this unconditional beta (applicable to the entire set of future possible returns) is related to the two conditional betas (that applicable if the covid scenario will arise and that applicable if it will not). Suppose that the expected returns in each of the two scenarios (covid and no covid, designated C and D) are equal, for both the market portfolio and asset x. So, $E_m = E_{mC} = E_{mD}$ and $E_x = E_{xC} = E_{xD}$. Equation (2) then becomes

$$\beta = \frac{\frac{.075}{N}\sum_{j=1}^{j=N}\left(R_{xj} - E_{xc}\right)\left(R_{mj} - E_{mc}\right) + \frac{.925}{N}\sum_{j=N+1}^{j=2N}(R_{xj} - E_{xD})(R_{mj} - E_{mD})}{\frac{.075}{N}\sum_{j=1}^{j=N}(R_{mj} - E_{mc})^2 + \frac{.925}{N}\sum_{j=N+1}^{j=2N}(R_{mj} - E_{mD})^2} \qquad (3)$$

Aside from the probabilities of 0.075 and 0.925, the two sets of terms in the denominator are the variances of the market returns under each of the two possible scenarios. Suppose these are equal, and designated V. Dividing top and bottom of the right hand side of equation (3) by V then yields the following:

$$\beta = \frac{\frac{0.075}{NV}\sum_{j=1}^{j=N}\left(R_{xj} - E_{xc}\right)\left(R_{mj} - E_{mc}\right) + \frac{0.925}{NV}\sum_{j=N+1}^{j=2N}(R_{xj} - E_{xD})(R_{mj} - E_{mD})}{0.075 + 0.925}$$

Aside from the probabilities of 0.075 and 0.925, the first set of terms in the numerator is the covid beta β_C, and the second set of terms in the numerator is the non-covid beta β_D. So, the unconditional beta β is a weighted average of the two conditional betas:

$$\beta = 0.075\beta_C + 0.925\beta_D \qquad (4)$$

This is the formula seemingly used by the Commerce Commission. Using the conditional betas of $\beta_C = 0.53$ and $\beta_D = 0.93$ invoked by the Commerce Commission, the unconditional beta would then be $\beta = 0.56$ as noted by the Commerce Commission (ibid, para 4.64). However, this equation (4) rests on two assumptions: that expected returns are equal in the covid and no-covid scenarios, and that the variance of the market returns is the same in both scenarios. By their very nature, covid type scenarios could be presumed to involve a higher variance of market returns. Consistent with this presumption, the variance of market returns was significantly elevated in the three-month period commencing on 1 March 2020.[1] So, the second assumption appears to be false. In addition, the covid scenario may involve a lower expected return, and therefore the first assumption may also be false.

To investigate the impact of higher variance under a covid scenario, suppose the covid scenario has a variance twice that of the no-covid scenario. Aside from the probabilities of 0.075 and 0.925, the two sets of terms in the denominator of equation (3) would then be $2V$ and V respectively, i.e.,

$$\beta = \frac{\frac{.075}{N}\sum_{j=1}^{j=N}\left(R_{xj} - E_{xc}\right)\left(R_{mj} - E_{mc}\right) + \frac{.925}{N}\sum_{j=N+1}^{j=2N}(R_{xj} - E_{xD})(R_{mj} - E_{mD})}{.075(2V) + .925V}$$

Dividing top and both of the right hand side of the last equation by V then yields

$$\beta = \frac{\frac{0.15}{N(2V)}\sum_{j=1}^{j=N}\left(R_{xj} - E_{xc}\right)\left(R_{mj} - E_{mc}\right) + \frac{0.925}{NV}\sum_{j=N+1}^{j=2N}(R_{xj} - E_{xD})(R_{mj} - E_{mD})}{0.15 + 0.925}$$

Aside from the numbers of 0.15 and 0.925 in the numerator, the first set of terms in the numerator is the covid beta β_C, and the second set of terms in the numerator is the non-covid beta β_D. So, the last equation is

$$\beta = \frac{0.15\beta_C + 0.925\beta_D}{0.15 + +0.925} = 0.14\beta_C + 0.86\beta_D$$

So, the weight on the covid beta would rise from 0.075 to 0.14. Using the conditional betas of β_C = 0.53 and β_D = 0.93 invoked by the Commerce Commission (ibid, para 4.64), the unconditional beta would then be β = 0.59 rather than 0.56. Thus, the weight on the covid beta depends not just on the probability of that scenario arising but also on the variance of market returns in the covid scenario relative to the non-covid scenario.

[1] Using the NZX 50 Gross Index, the standard deviation of the daily returns in this three month period was four times that in the preceding three years.

An alternative approach to the Commission's approach of weighting the conditional betas would be to apply weights to the covid and non-covid returns data, and then generate a single estimate of beta. This is the empirical counterpart to equation (2), with average returns for asset *x* and the market portfolio being used rather than their expectations. Such an approach does not require the assumptions that underlie the Commission's approach. Interestingly, the Commerce Commission (ibid, para 4.58 – 4.62) refers to work by Flint (2021) and TDB (2023), and seems to characterize their work as being of the type in equation (4), i.e., weighting over beta estimates. However, TDB's work instead seems to involve weighting returns data from both covid and non-covid scenarios within the beta estimate rather than weighting betas, as in equation (2), and therefore avoids the problem identified here with the use of equation (4). Furthermore, whilst Flint (2021, Table 6) does present results from the same approach as equation (4), their preferred method shown in their Table 6 is to weight returns data from both covid and non-covid scenarios within the beta estimate, as in equation (2), which again avoids the problem identified here with the use of equation (4). Furthermore, Flint's (2021, Table 6) results from their preferred approach are significantly higher than from equation (4), and therefore the analysis in the current paper could then be viewed as explaining why this difference arises.

3. Conclusions

In a recent draft report on WACC, the Commerce Commission concluded that covid-19 raised the betas of airports and there may be a material chance of a future event of that type, leading to them estimating the beta relevant to a future regulatory period by using data from both the pre covid and covid periods. The approach used by the Commission involves estimating betas for each of the two scenarios and then weighting the two betas by the probabilities of each scenario arising. An alternative approach is to weight the returns data using these probabilities, and then generate one estimate of the beta. The second approach is consistent with the definition of beta whilst the first approach will only be consistent under conditions that do not seem to apply. Accordingly, if allowance for a future covid scenario is to be done, the second approach would seem to be preferable.

References

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