The Logic and Economics underlying the use of a 75% Rule in a Regulatory Environment

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I. Introduction

1. A 75% Rule is a regulatory rule whereby each individual parameter estimate used in the determination of the cost of capital is set at a level different from the expected value of the parameter. The selected level is such that, assuming that all other parameters are measured correctly, then only 25% of the time is the use of the selected parameter estimate associated with an underestimate of the cost of capital. This note considers four issues. Section II explains why it is optimal to employ such a rule even when investment has already occurred. Section III sets out a further reason to employ such a rule when the level of investment is not fixed, but is a choice to be made by regulated firms. Section IV explains the need for time consistency in employing a 75% rule and shows why that the further reason also applies to existing investments. Section V discusses how a regulator might go about selecting the optimal rule to employ; i.e., how a regulator might go about choosing between a 75% rule and an 80% rule.

II. The Rationale for a 75% Rule when Investment is Fixed

1. A regulated business faces risks for which it is not explicitly compensated. One such risk is that technological advances in substitute products can have the effect that the market for the regulated business's product shrinks dramatically and its assets become stranded. When a regulators' estimate of future profits assigns no probability to stranding risk, the regulators' estimate of future profits overstates the true expected

profit and a regulated business cannot expect to earn a normal return unless the regulatory building blocks somehow compensate for that risk elsewhere. One way of doing so is to set the allowed rate of return above the cost of capital.

2. A second such risk arises whenever a regulated entity faces the risk of a natural disaster (e.g. an earthquake) that is not recognized in the regulator's estimate of future profits. Again, the regulator's overestimate of future profits can be offset by an adjustment that sets the allowed rate of return above the cost of capital.

3. One can think of a 75% rule as a fudge factor to recognize the existence of stranding risk and disaster risk. A better approach would be to estimate the expected cost associated with stranding and natural disasters and compensate for what is in effect the cost borne by the regulated firm in self-insuring against such risks.

4. A further rationale for a 75% rule is as way of recognizing the existence of the bias induced by the regulatory underestimation of expected operating costs. The estimate of future profits used by a regulator will exceed the true value of a regulated business's expected future profits. This occurs whenever demand is uncertain and marginal cost is increasing.

5. The regulator's calculation of the cost of producing the expected quantity demanded at the allowed price is an underestimate of the true expected cost of producing the quantity demanded at the allowed price. To see this, suppose a regulated company is has invested the amount I and assume for simplicity that the investment will generate profits in perpetuity. Assume that the quantity demanded is a random function of the price of the form $\tilde{q}(p) = \tilde{a} + \tilde{b}p$, where $\tilde{q}(p)$ is the random quantity demanded given a price of p and $\tilde{a}$ and $\tilde{b}$ are random parameters. A tilde, $\tilde{\bullet}$, above a parameter denotes that parameter is random. In this case the intercept and the slope of the

demand function are random variables. The quantity demanded is decreasing in price and hence the slope parameter $\tilde{b}$ of the demand function is negative.

6. The expected quantity demanded at a price of $\hat{p}$ is

$$E\{\tilde{q}(p)|p=\hat{p}\}=E\{\tilde{a}+\tilde{b}\hat{p}\}=E\{\tilde{a}\}+E\{\tilde{b}\}\hat{p}.$$

7. The cost of producing q units is a random function $\tilde{c}(q)=\tilde{x}+\tilde{y}q+\tilde{z}q^2$ where $\tilde{x}$, $\tilde{y}$ and $\tilde{z}$ are random parameters. The cost function is such that the marginal cost of production, $\frac{d\tilde{c}(q)}{dq}=\tilde{y}+2\tilde{z}q$, is increasing in the quantity produced; i.e.,

$$\frac{d\left(d\tilde{c}(q)/dq\right)}{dq}=\frac{d^2\tilde{c}(q)}{dq^2}=2\tilde{z}>0.$$

8. A regulator sets the allowed price for the company's product by assuming that the actual quantity demanded is equal to the quantity expected to be demanded at the allowed price. The expected cost of producing a quantity equal to the quantity that is expected to be sold at a price of $\hat{p}$ is given by $E\{\tilde{c}(q)|q=E\{\tilde{q}(\hat{p})\}\}$.

Regulator's Estimate of Expected Profit: $E\{\tilde{q}(\hat{p})\}\hat{p}-E\{\tilde{c}(q)|q=E\{\tilde{q}(\hat{p})\}\}$.

9. The regulator's estimate of expected profits is an overestimate of the true value of the expected profit given the allowed price. Because the cost of production is a convex function of the quantity produced and the quantity sold at a price of $\hat{p}$ is random, the regulator's estimate of expected profit will be *greater* than the true value of the expected profit at a price of $\hat{p}$. The true value of the expected profit given the allowed price is

True Value of Expected Profit: $E\{\tilde{q}(\hat{p})\}\hat{p}-E\{\tilde{c}(\tilde{q}(\hat{p}))\}$.

10. Let $\hat{k}$ denote an unbiased estimate of the true value of the cost of capital. Suppose the regulator does not employ a 75% rule and sets the allowed return equal to an unbiased estimate of the true value of the cost of capital. The regulator will set the allowed price equal to $p*$ where $p*$ solves

$$E\{\tilde{q}(p*)\}\, p* \;-\; E\left\{\tilde{c}\left(E\{\tilde{q}(p*)\}\right)\right\} \;=\; \hat{k}I.$$

$$E\left\{\left(\tilde{a}+\tilde{b}p*\right)\right\} p* - \left(E\{\tilde{x}\} + E\{\tilde{y}\}\left[E\left\{\left(\tilde{a}+\tilde{b}p*\right)\right\}\right] + E\{z\}\left[E\left\{\left(\tilde{a}+\tilde{b}p*\right)\right\}\right]^2\right) = \hat{k}I. \quad (1)$$

11. At the allowed price of $p*$ the true value of the expected profit is actually

$$E\{\tilde{q}(p*)\}\, p* - E\left\{\tilde{c}\left(\tilde{q}(p*)\right)\right\} =$$

$$E\left\{\left(\tilde{a}-\tilde{b}p*\right)\right\} p* - \left(E\{\tilde{x}\} + E\{\tilde{y}\}\, E\left\{\left(\tilde{a}-\tilde{b}p*\right)\right\} + E\{\tilde{z}\}\, E\left\{\left(\tilde{a}-\tilde{b}p*\right)^2\right\}\right).$$

12. Given (1), the *true* value of the expected profit can be rewritten as

$$\begin{aligned}
&\hat{k}I + E\{\tilde{z}\}\left[E\left\{\left(\tilde{a}-\tilde{b}p*\right)\right\}\right]^2 - E\{\tilde{z}\}\, E\left\{\left(\tilde{a}-\tilde{b}p*\right)^2\right\} \\
&= \hat{k}I + E\{\tilde{z}\}\left[\left[E\{\tilde{a}\}\right]^2 - E\left\{\tilde{a}^2\right\} + \left(\left[E\left\{\tilde{b}\right\}\right]^2 - E\left\{\tilde{b}^2\right\}\right)[p*]^2\right] \\
&= \hat{k}I - E\{\tilde{z}\}\left[Var(\tilde{a}) + Var\left(\tilde{b}\right)[p*]^2\right],
\end{aligned}$$

which is less than $\hat{k}I$; i.e., less than the regulator's estimate of the expected profit. The true value is less than the regulator's estimate whenever (i) $E\{\tilde{z}\}$ is positive; i.e., whenever the firm faces an increasing marginal costs of production, and (ii) the firm faces a random demand for its product; i.e., $Var(\tilde{a})$ and/or $Var\left(\tilde{b}\right)$ are positive.

13. In practice the quantity demanded is random and the marginal operating cost is an increasing function of quantity. Therefore, it is natural to set the allowed rate of return above an unbiased estimate of the cost of capital and one can think of a 75% rule as an attempt to offset the upward bias in the regulatory estimate of expected profits due

to increasing marginal operating costs as well as the regulatory failure to recognize stranding and disaster risk.

III. A Further Rationale for a 75% Rule when Investment is Not Fixed

1. Suppose that because of standing risk, disaster risk, and the effect of demand uncertainty and increasing marginal costs, the allowed rate of return is set at a level above an unbiased estimate of the cost of capital. The regulator can then set an allowed price with the property that the true expected return is an unbiased estimate of the cost of capital. This does *not* mean that potential investors will be willing to invest the amount I in the regulated business. Rather, it means that if it were compulsory to invest the amount I in the regulated business then on average investors would earn the cost of capital.

2. In general, potential investors are not required to invest in regulated businesses.[1] Potential investors will only invest when the true expected return from doing so is greater than or equal to the cost of capital. Thus, if (i) the regulated price is set at a level such that the expected value of the true rate of the return that investors will earn on their investment is equal to an unbiased estimate of investors' true required return, and (ii) uncertainty is symmetric, then only 50% of the time will the expected value of the rate of the return that investors will earn on their investment exceed investors' true required return. In the other 50% of cases investors' true required return will exceed the regulator's unbiased estimate of investors' required return and investors will not be willing to invest.

[1] Moreover, even if compulsory investment might be able to be elicited from some investors in some circumstances, relying heavily on such a mechanism in those circumstances would likely increase the promised return demanded by investors before making future investments that might lead to further compulsory investments.

3. If a regulator wanted to be certain that investment would always take place, the regulator would have to allow a price such that the expected value of the rate of return that investors will actually earn on their investment was equal to the maximum possible value of investors' required return. Note that given standing risk, disaster risk, and the effect of demand uncertainty and increasing marginal costs, investment could only be guaranteed if a *greater than* 100% rule were used.

IV. Time Consistency and the 75% Rule

1. When an investment has already been made, a regulator might reason that it is sufficient to set the allowed price at a level such that the true expected return in the future is an unbiased estimate of the regulated firm's cost of capital. Such reasoning is not consistent with using a Percent Rule so as to actually achieve the desired likelihood of investment. Investors will anticipate the future downward revision in the estimated cost of capital and will therefore be less willing to initially invest.

2. Switching from a 75% rule will accomplish a one-time redistribution of wealth away from the owners of the regulated business. And doing so will simultaneously diminish the regulator's credibility with future investors and therefore diminish the regulator's ability to achieve their goals in their future regulatory endeavours.

V. What % Rule Should a Regulator Employ?

1. To answer this question the regulator must first explicitly determine the loss function they are seeking to minimise. The regulator must determine the rate at which they are willing to trade off the economic loss associated with underinvestment in

infrastructure against any economic loss associated with greater than normal returns to investors in regulated businesses.

2. Given the uncertainty in demand and cost parameters and the estimation error in the parameters used to operationalize the capital asset pricing model, a regulator can use a bootstrap technique to determine the likelihood that a profit-maximizing regulated business will find it optimal to invest given a 75% rule.[2] A bootstrap approach would estimate the probability of investing by repeated sampling of the parameters from the empirical distribution of the observed data and asking whether the net present value is positive in each simulation. The percentage of simulations with positive NPV's will give the likelihood that a profit-maximizing regulated business would elect to invest under a 75% rule.

3. Similarly, the regulator could determine the likelihood that a profit-maximizing regulated business would elect to invest under an 80% rule, etc. The regulator would then have to determine what risk of underinvestment they were willing to bear and select the % rule that is optimal given their loss function.

4. Trying to solve the problem analytically is very difficult because an analytical solution requires an explicit determination of the multivariate distribution of the set of parameters describing the demand and cost functions as well as the parameters of the asset pricing model that determine the regulator's estimate of the cost of capital.[3] It is difficult to have an intuitive sense of the analytical probability that investment will

[2] For an introduction to the bootstrap technique see Efron, Bradley and R.J. Tibshirani, 1994, "An Introduction to the Bootstrap" (Chapman & Hall/CRC Monographs on Statistics & Applied Probability).

[3] The distribution used in the bootstrap approach is the empirical observed distribution of the parameters.

incur under a 75% rule. For example, consider a simple setting with just two unknown parameters *x* and *y* and suppose that an estimator of the cost of capital is given by the sum of *x* and *y*. Suppose also that one has implemented a 75% rule by adding together two estimators $\hat{x}$ and $\hat{y}$ each with the property that there is only a 25% chance that the true value of the parameter actually exceeds the estimator of the parameter. Thus there is a 25% chance that the true value of *x* actually exceeds $\hat{x}$ and a 25% chance that the true value of *y* actually exceeds $\hat{y}$.

5. What is the probability that the true value of the sum $x+y$ actually exceeds the sum of the estimators $\hat{x}+\hat{y}$? One might guess that the probability is quite low and equal to $0.25 \times 0.25 =$ 6.25%. The actual probability that $x+y>\hat{x}+\hat{y}$ depends on the multivariate distribution of *x* and *y*. Suppose *x* and *y* are independent and normally distributed. Suppose also that the two variables have identical variances. In that event, the probability that the true value of $x+y>\hat{x}+\hat{y}$ is 17%. The risk that firms will not invest is 17%, not 6.25%.